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Transient Thermomechanical Stresses of Functionally Graded Cylindrical Panels

Z. S. Shao* and T. J. Wang†
Xi'an Jiaotong University, 710049 Xi'an, People's Republic of China
and
K. K. Ang‡
National University of Singapore, Kent Ridge 119260, Republic of Singapore

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Three-dimensional thermomechanical analysis of functionally graded cylindrical panels subjected to nonuniform mechanical and transient thermal loads is carried out in this paper. Thermal and mechanical properties are assumed as temperature-independent and continuously varying in the radial direction. Employing Laplace transform and the series solving method of ordinary differential equation(s), solutions of time-dependent temperature and thermomechanical stress in functionally graded material cylindrical panels are obtained. As an example, a functionally graded material cylindrical panel composed of molybdenum and mullite is calculated and all results are graphically presented.

Nomenclature

$e_r, e_\theta, e_z, e_{r\theta}, e_{\theta z}, e_{rz}$ = dimensionless strain components
H_1, H_2 = dimensionless heat transfer coefficients
h_1, h_2 = heat transfer coefficients on the inner and outer surfaces
L = dimensionless axial length
l = axial length of the cylindrical panel
Q_a, Q_b = dimensionless internal and external pressures
q_a, q_b = internal and external pressures
R, Z = dimensionless coordinates
R_a, R_b = dimensionless inner and outer radii
r, θ, z = cylindrical coordinates
r_a, r_b, r_m = inner, outer, and average radii
T = temperature
T_a, T_b = temperature of internal and external media
$T_0, E_0, \alpha_0, \lambda_0, \kappa_0$ = reference temperature, elastic modulus, thermal expansion, conductivity, and diffusivity
U, V, W = dimensionless displacement components
u, v, w = displacement components
Y = dimensionless elastic modulus
α, λ, κ = thermal expansion, conductivity, and diffusivity
$\varepsilon_r, \varepsilon_\theta, \varepsilon_z, \gamma_{r\theta}, \gamma_{\theta z}, \gamma_{rz}$ = strain components
Θ = dimensionless temperature
Θ_a, Θ_b = dimensionless temperature of internal and external media
μ, E = Poisson's ratio and elastic modulus
$\Sigma_r, \Sigma_\theta, \Sigma_z, \Sigma_{r\theta}, \Sigma_{\theta z}, \Sigma_{rz}$ = dimensionless stress components
$\sigma_r, \sigma_\theta, \sigma_z, \sigma_{r\theta}, \sigma_{\theta z}, \sigma_{rz}$ = stress components
Ω, Λ, K = dimensionless thermal expansion, conductivity, and diffusivity

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*Ph.D., Ministry of Education Key Laboratory for Strength and Vibration; zhushan.shao@gmail.com.

†Professor, Ministry of Education Key Lab for Strength and Vibration; wangtj@mail.xjtu.edu.cn.

‡Associate Professor, Department of Civil Engineering; cveangkk@nus.edu.sg.

Introduction

FUNCTIONALLY graded materials (FGMs) are those in which the volume fractions of two or more materials are varied continuously as a function of position along certain direction(s) of the structure to achieve a required function. For example, thermal barrier plate structures for high-temperature applications may be formed from a mixture of ceramic and a metal. The composition is varied from a ceramic-rich surface to a metal-rich surface, with a desired variation of the volume fractions of the two materials between the two surfaces. The ceramic constituent of the material provides the high temperature resistance due to its low thermal conductivity. The ductile metal constituent, on the other hand, prevents fracture caused by stresses due to high temperature gradients in a very short period of time. The gradual change of material properties can be tailored to different applications and working environments. This makes functionally graded materials preferable in many applications.

FGMs are expected to be used for high-temperature structures such as turbine blades [1]. If transient thermomechanical stresses that arise during heating or cooling of a structure are severe enough, fracture may occur. So the thermomechanical problems of FGM structures have attracted the attention of many researchers in the past 20 years. Noda [2] presented an extensive review that covers a wide range of topics, from thermoelastic to thermoinelastic problems. He discussed the effect of temperature-dependent mechanical properties on stresses and suggested that temperature-dependent properties of the material be taken into account to perform more accurate analysis. Tanigawa [3] compiled a comprehensive list of papers on the analytical models of thermoelastic behavior of functionally graded materials. Obata and Noda [4] considered steady-state thermal stresses in a hollow circular cylinder and a hollow sphere made of a FGM to understand the effect of the volumetric ratio of constituents and porosity on thermal stresses. They discussed the design of an optimum functionally graded material by minimizing the thermal stresses. Reddy and his coworkers [5–7] carried out theoretical and finite element analyses of the thermomechanical behavior of functionally graded cylinders, plates, and shells. In their works, geometric nonlinearity and the effect of the coupling item was considered for different thermal loading conditions. Batra and Vel [8,9] studied the thermomechanical deformations of thick functionally graded plates. Based on a higher-order shear plate theory, transient deformations were derived by using the meshless local Petrov–Galerkin method, and analytical solutions were obtained by using the Laplace transformation technique and the power-series method.

Because of the nonhomogeneity of FGM and loading and the geometric complexity of a problem, it is not easy to obtain analytical solutions for the temperature and stresses in FGM structures. Occasionally, if the material model used is simple and the geometry is regular, one can obtain the analytical solution for the thermomechanical problems of FGM structures. Based on the assumption that the material properties of FGM obey power law, Shao and Wang [10] theoretically analyzed the steady-state thermomechanical stresses of a FGM cylindrical panel by using the Frobinus method. Furthermore, Shao et al. [11] employed an exponential model and theoretically studied the transient thermomechanical stresses of a functionally graded circular hollow cylinder, in which the inverse of Laplace transformation was theoretically carried out by using the Galerkin method. Ootao and Tanigawa [12] obtained the analytical solutions of thermomechanical stress for a FGM cylindrical panel. Jabbari and his coworkers [13] obtained the analytical solutions for a two-dimensional steady-state thermomechanical problem of a FGM circular hollow cylinder. Using the Fourier series, Pan and Roy [14] carried out the analytical solution of static deformations of a multilayered functionally graded elastic cylinder. A widely used approximate theoretically method is the multilayered method, in which each layer is assumed to be homogeneous. Continuous conditions between layers are used to develop the final solution of the problem. This method has been employed by many researchers to study the thermomechanical problems of FGM structures such as FGM cylinders [15,16].

In present work, we consider the three-dimensional transient thermomechanical problem of a functionally graded cylindrical panel. The panel is simply supported at its four end surfaces and subjected to nonuniform mechanical and transient thermal loads on its inner and outer surfaces, respectively. Material properties of the panel are assumed to be temperature-independent and varying continuously in the radial direction. The Laplace transform and series solving method of ordinary differential equation(s) are employed to derive the three-dimensional solutions of time-dependent temperature and transient thermomechanical stresses in the FGM panel.

Transient Thermomechanical Analysis

We consider a functionally graded cylindrical panel, shown in Fig. 1. The panel is simply supported on its four end edges and subjected to nonuniform internal pressure and external pressure on the inner and outer surfaces. Its initial temperature is zero and suddenly heated by surrounding media on the inner and outer surfaces. The temperatures at four end edges remain at zero.

Generally, Poisson's ratio of materials varies in a small range. For simplicity, we assume μ of FGM to be a constant. Moreover, we assume that elastic modulus, thermal expansion, thermal conductivity, and thermal diffusivity change smoothly and continuously through the thickness of the panel.

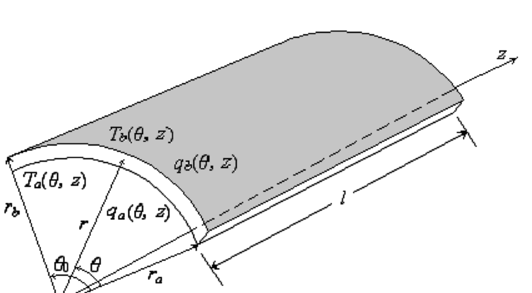

Fig. 1 Dimensions and loading conditions of functionally graded cylindrical panel with coordinate system.

For the sake of simplicity, dimensionless variables will be used in the following derivative process. And dimensionless variables are defined as

$$R = r/r_m, \quad Z = z/r_m, \quad R_a = r_a/r_m, \quad R_b = r_b/r_m$$
$$L = l/r_m, \quad Y = E/E_0, \quad \Omega = \alpha/\alpha_0, \quad \Lambda = \lambda/\lambda_0$$
$$K = \kappa/\kappa_0, \quad \tau = (\kappa_0/r_m^2)t, \quad \Theta = T/T_0, \quad \Theta_a = T_a/T_0$$
$$\Theta_b = T_b/T_0, \quad H_1 = r_m h_1, \quad H_2 = r_m h_2$$
$$U = u/\alpha_0 T_0 r_m, \quad W = w/\alpha_0 T_0 r_m, \quad e_{rr} = \varepsilon_r/(\alpha_0 T_0)$$
$$e_{\theta\theta} = \varepsilon_\theta/(\alpha_0 T_0), \quad e_{zz} = \varepsilon_z/(\alpha_0 T_0), \quad e_{r\theta} = \gamma_{r\theta}/(\alpha_0 T_0)$$
$$e_{\theta z} = \gamma_{\theta z}/(\alpha_0 T_0), \quad e_{rz} = \varepsilon_{rz}/(\alpha_0 T_0), \quad \Sigma_r = \sigma_r/(\alpha_0 E_0 T_0)$$
$$\Sigma_\theta = \sigma_\theta/(\alpha_0 E_0 T_0), \quad \Sigma_z = \sigma_z/(\alpha_0 E_0 T_0)$$
$$\Sigma_{r\theta} = \sigma_{r\theta}/(\alpha_0 E_0 T_0), \quad \Sigma_{\theta z} = \sigma_{\theta z}/(\alpha_0 E_0 T_0)$$
$$\Sigma_{rz} = \sigma_{rz}/(\alpha_0 E_0 T_0), \quad Q_a = q_a/(\alpha_0 E_0 T_0)$$
$$Q_b = q_b/(\alpha_0 E_0 T_0)$$

Basic Equations

For the present problem, the transient heat conduction equation expressed by using the dimensionless variables can be written as

$$\left[\frac{\partial^2}{\partial R^2} + \left(\frac{1}{\Lambda(R)}\frac{\mathrm{d}\Lambda(R)}{\mathrm{d}R} + \frac{1}{R}\right)\frac{\partial}{\partial R} + \frac{1}{R^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial Z^2}\right]\Theta = \frac{1}{K(R)}\frac{\partial \Theta}{\partial \tau} \tag{1}$$

And initial-condition temperature boundary conditions can be written as

$$\Theta(R, \theta, Z, 0) = 0 \tag{2}$$

$$\Theta(R, 0, Z, \tau) = \Theta(R, \theta_0, Z, \tau) = 0 \tag{3a}$$

$$\Theta(R, \theta, 0, \tau) = \Theta(R, \theta, L, \tau) = 0 \tag{3b}$$

$$\frac{\partial \Theta(R_a, \theta, Z, \tau)}{\partial R} - H_1 \Theta(R_a, \theta, Z, \tau) = -H_1 \Theta_a(\theta, Z) \tag{4a}$$

$$\frac{\partial \Theta(R_b, \theta, Z, \tau)}{\partial R} + H_2 \Theta(R_b, \theta, Z, \tau) = H_2 \Theta_b(\theta, Z) \tag{4b}$$

Considering dimensionless variables already defined, for the present thermomechanical problem, the constitutive equations can be written as

$$\Sigma_r = \frac{Y(r)}{(1+\mu)(1-2\mu)}[(1-\mu)e_r + \mu e_\theta + \mu e_z] + \frac{\Omega(r)\cdot Y(r)}{1-2\mu}\Theta \tag{5a}$$

$$\Sigma_\theta = \frac{Y(r)}{(1+\mu)(1-2\mu)}[\mu e_r + (1-\mu)e_\theta + \mu e_z] + \frac{\Omega(r)\cdot Y(r)}{1-2\mu}\Theta \tag{5b}$$

$$\Sigma_z = \frac{Y(r)}{(1+\mu)(1-2\mu)}[\mu e_r + \mu e_\theta + (1-\mu)e_z] + \frac{\Omega(r)\cdot Y(r)}{1-2\mu}\Theta \tag{5c}$$

$$\Sigma_{\theta z} = \frac{Y(r)}{2(1+\mu)} e_{\theta z}, \qquad \Omega_{zr} = \frac{Y(r)}{2(1+\mu)} e_{zr}$$
$$\Sigma_{r\theta} = \frac{Y(r)}{2(1+\mu)} e_{r\theta} \tag{5d}$$

The geometric equations can be written as

$$e_r = \frac{\partial U}{\partial R}, \qquad e_\theta = \frac{1}{R}\left(\frac{\partial V}{\partial \theta} + U\right), \qquad e_z = \frac{\partial W}{\partial Z} \tag{6a}$$

$$e_{r\theta} = \frac{1}{R}\frac{\partial U}{\partial \theta} + \frac{\partial V}{\partial R} - \frac{V}{R}, \qquad e_{\theta z} = \frac{\partial V}{\partial Z} + \frac{1}{R}\frac{\partial W}{\partial \theta}$$
$$e_{zr} = \frac{\partial W}{\partial R} + \frac{\partial U}{\partial Z} \tag{6b}$$

And the equilibrium equations can be written as

$$\frac{\partial \Sigma_r}{\partial R} + \frac{1}{R}\frac{\partial \Sigma_{r\theta}}{\partial \theta} + \frac{\partial \Sigma_{zr}}{\partial Z} + \frac{\Sigma_r - \Sigma_\theta}{R} = 0 \tag{7a}$$

$$\frac{\partial \Sigma_{r\theta}}{\partial R} + \frac{1}{R}\frac{\partial \Sigma_\theta}{\partial \theta} + \frac{\partial \Sigma_{\theta z}}{\partial Z} + \frac{2\Sigma_{r\theta}}{R} = 0 \tag{7b}$$

$$\frac{\partial \Sigma_{rz}}{\partial R} + \frac{1}{R}\frac{\partial \Sigma_{\theta z}}{\partial \theta} + \frac{\partial \Sigma_z}{\partial Z} + \frac{\Sigma_{rz}}{R} = 0 \tag{7c}$$

For the present thermomechanical problem, boundary conditions of displacement and stress can be expressed as

$$\left.\begin{array}{c} U(R,0,Z,\tau) = W(R,0,Z,\tau) = 0 \\ \Sigma_\theta(R,0,Z,\tau) = \Sigma_{r\theta}(R,0,Z,\tau) = \Sigma_{\theta z}(R,0,Z,\tau) = 0 \end{array}\right\} \tag{8a}$$

$$\left.\begin{array}{c} U(R,\theta_0,Z,\tau) = W(R,\theta_0,Z,\tau) = 0 \\ \Sigma_\theta(R,\theta_0,Z,\tau) = \Sigma_{r\theta}(R,\theta_0,Z,\tau) = \Sigma_{\theta z}(R,\theta_0,Z,\tau) = 0 \end{array}\right\} \tag{8b}$$

$$\left.\begin{array}{c} U(R,\theta,0,\tau) = V(R,\theta,0,\tau) = 0 \\ \Sigma_z(R,\theta,0,\tau) = \Sigma_{rz}(R,\theta,0,\tau) = \Sigma_{\theta z}(R,\theta,0,\tau) = 0 \end{array}\right\} \tag{8c}$$

$$\left.\begin{array}{c} U(R,\theta,L,\tau) = V(R,\theta,L,\tau) = 0 \\ \Sigma_z(R,\theta,L,\tau) = \Sigma_{rz}(R,\theta,L,\tau) = \Sigma_{\theta z}(R,\theta,L,\tau) = 0 \end{array}\right\} \tag{8d}$$

$$\begin{array}{l} \Sigma_r(R_a,\theta,Z,\tau) = Q_a(\theta,Z,\tau), \\ \Sigma_{rz}(R_a,\theta,Z,\tau) = \Sigma_{r\theta}(R_a,\theta,Z,\tau) = 0 \end{array} \tag{9a}$$

$$\begin{array}{l} \Sigma_r(R_b,\theta,Z,\tau) = Q_b(\theta,Z,\tau), \\ \Sigma_{rz}(R_b,\theta,Z,\tau) = \Sigma_{r\theta}(R_b,\theta,Z,\tau) = 0 \end{array} \tag{9b}$$

Substituting Eqs. (5) and (6) into Eq. (7), we have

$$\begin{aligned} &\left[\frac{\partial^2}{\partial R^2} + \left(\frac{Y'(R)}{Y(R)} + \frac{1}{R}\right)\frac{\partial}{\partial R} + \left(\frac{\mu}{1-\mu}\frac{1}{R}\frac{Y'(R)}{Y(R)} - \frac{1}{R^2}\right)\right. \\ &\left. + \frac{1-2\mu}{2-2\mu}\frac{\partial^2}{\partial \theta^2} + \frac{1-2\mu}{2-2\mu}\frac{\partial^2}{\partial Z^2}\right]U \\ &+ \left[\frac{1}{2-2\mu}\frac{1}{R}\frac{\partial^2}{\partial R\partial\theta} + \frac{\mu}{1-\mu}\frac{1}{R}\frac{Y'(R)}{Y(R)}\frac{\partial}{\partial\theta} + \frac{4\mu-3}{2-2\mu}\frac{1}{R^2}\frac{\partial}{\partial\theta}\right]V \\ &+ \left[\frac{1}{2-2\mu}\frac{\partial^2}{\partial R\partial Z} + \frac{\mu}{1-\mu}\frac{Y'(R)}{Y(R)}\frac{\partial}{\partial Z}\right]W \\ &+ \frac{1+\mu}{1-\mu}\left[\Omega(R)\cdot\frac{\partial}{\partial R} + \frac{[Y(R)\cdot\Omega(R)]'}{Y(R)}\right]\Theta = 0 \end{aligned} \tag{10a}$$

$$\begin{aligned} &\left[\frac{1}{1-2\mu}\frac{1}{R}\frac{\partial^2}{\partial R\partial\theta} + \left(\frac{1}{R}\frac{Y'(R)}{Y(R)} + \frac{3-4\mu}{1-2\mu}\frac{1}{R^2}\right)\frac{\partial}{\partial\theta}\right]U \\ &+ \left[\frac{\partial^2}{\partial R^2} + \left(\frac{Y'(R)}{Y(R)} + \frac{1}{R}\right)\frac{1}{R}\frac{\partial}{\partial R} - \frac{1}{R}\frac{Y'(R)}{Y(R)}\right. \\ &\left. - \frac{1}{R^2} + \frac{2-2\mu}{1-2\mu}\frac{1}{R^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial Z^2}\right]V + \frac{1}{1-2\mu}\frac{1}{R}\frac{\partial^2 W}{\partial\theta\partial Z} \\ &+ \frac{2+2\mu}{1-2\mu}\Omega(R)\cdot\frac{1}{R}\frac{\partial\Theta}{\partial\theta} = 0 \end{aligned} \tag{10b}$$

$$\begin{aligned} &\left[\frac{1}{1-2\mu}\frac{\partial^2}{\partial R\partial Z} + \left(\frac{Y'(R)}{Y(R)} + \frac{1}{1-2\mu}\frac{1}{R}\right)\frac{\partial}{\partial Z}\right]U + \frac{1}{1-2\mu}\frac{1}{R}\frac{\partial^2 V}{\partial\theta\partial Z} \\ &+ \left[\frac{\partial^2}{\partial R^2} + \left(\frac{Y'(R)}{Y(R)} + \frac{1}{R}\right)\frac{\partial}{\partial R} + \frac{1}{R^2}\frac{\partial^2}{\partial\theta^2} + \frac{2-2\mu}{1-2\mu}\frac{\partial^2}{\partial Z^2}\right]W \\ &+ \frac{2+2\mu}{1-\mu}\Omega(R)\cdot\frac{\partial\Theta}{\partial Z} = 0 \end{aligned} \tag{10c}$$

Now a three-dimensional thermomechanical problem is formulated. In what follows, the Laplace transform and series solving method of ordinary differential equation(s) will be employed to solve the thermomechanical problem.

Analysis of Time-Dependent Temperature

Employing a Navier trigonometric series, solution of Eq. (1) satisfying the temperature boundary conditions (3) can be assumed as

$$\Theta(R,\theta,Z,\tau) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\Theta_{nm}(R,\tau)\cdot\sin(b\theta)\sin(aZ) \tag{11}$$

where Θ_{nm} (R, τ) are unknown functions, $a = n\pi/L$, and $b = m\pi/\theta_0$. Substituting Eq. (11) into Eqs. (1), (2), and (4), respectively, we can obtain

$$\begin{aligned} &\left[\frac{\partial^2}{\partial R^2} + \left(\frac{1}{\Lambda(R)}\frac{\mathrm{d}\Lambda(R)}{\mathrm{d}R} + \frac{1}{R}\right)\frac{\partial}{\partial R} - \left(\frac{b^2}{R^2} + a^2\right)\right]\Theta_{nm}(R,\tau) \\ &= \frac{1}{\mathrm{K}(R)}\frac{\partial\Theta_{nm}(R,\tau)}{\partial\tau} \end{aligned} \tag{12}$$

and

$$\Theta_{nm}(R,0) = 0 \tag{13}$$

$$\frac{\partial\Theta_{nm}(R_a,\tau)}{\partial R} - H_a\Theta_{nm}(R_a,\tau) = -H_1\Theta_{anm} \tag{14a}$$

$$\frac{\partial \Theta_{nm}(R_b,\tau)}{\partial R} + H_2\Theta_{nm}(R_b,\tau) = H_2 T_{bnm} \tag{14b}$$

where

$$\Theta_{anm} = \frac{4}{\theta_0 L}\int_0^L\int_0^{\theta_0}\Theta_a(\theta,Z)\sin(b\theta)\sin(aZ)\,\mathrm{d}\theta\,\mathrm{d}Z$$
$$\Theta_{bnm} = \frac{4}{\theta_0 L}\int_0^L\int_0^{\theta_0}\Theta_b(\theta,z)\sin(b\theta)\sin(aZ)\,\mathrm{d}\theta\,\mathrm{d}Z$$

Considering initial condition (13), we apply the Laplace transform to Eq. (12) and boundary conditions (14) with respect to variable τ:

$$R^2\frac{\partial^2 F}{\partial R^2} + \left(R^2\frac{1}{\Lambda(R)}\frac{\mathrm{d}\Lambda(R)}{\mathrm{d}R} + R\right)\frac{\partial F}{\partial R} - \left(s^2\frac{R^2}{\mathrm{K}} - a^2R^2 - b^2\right)F = 0 \tag{15}$$

$$\frac{\partial F(R_a,s)}{\partial R} - H_a F(R_a,s) = -\frac{H_1\Theta_{anm}}{s} \tag{16a}$$

$$\frac{\partial F(R_b,s)}{\partial R} + H_2 F(R_b,s) = \frac{H_2\Theta_{bnm}}{s} \tag{16b}$$

where

$$F(R,s) = \angle[\Theta_{nm}(R,\tau)]$$

According to the series method of ordinary differential equations [17], if the coefficients $\Lambda'(R)/\Lambda(R)$ and $1/\mathrm{K}(R)$ are analytical at point $R = 1$ and could be expressed as Taylor's series in terms of $R - 1$, then the solution of Eq. (15) can be also expressed as the following Taylor's series:

$$F(R,s) = \sum_{k=0}^{\infty} A_k(s)\cdot(R-1)^k \tag{17}$$

Also, functions $\Lambda'(R)/\Lambda(R)$ and $1/\mathrm{K}(R)$ are expanded in Taylor's series at point $R = 1$:

$$f_1(R) = \frac{1}{\Lambda(R)}\frac{\mathrm{d}\Lambda(R)}{\mathrm{d}R} = \sum_{k=0}^{\infty} f_{1k}(r-1)^k \tag{18a}$$

$$f_2(R) = \frac{1}{\mathrm{K}(R)} = \sum_{k=0}^{\infty} f_{2k}(r-1)^k \tag{18b}$$

where

$$f_{1k} = \frac{1}{k!}f_1^{(k)}(1), \qquad f_{2k} = \frac{1}{k!}f_2^{(k)}(1)$$

Substituting Eqs. (17) and (18) into Eq. (15), employing the Abel principle of series multiplication, and comparing the coefficients of $(R-1)^k$, one then obtains the following recurrence equation:

$$\begin{aligned}(k+1)(k+2)A_{k+2} &= -(2k+1)(k+1)A_{k+1} + (a^2+b^2-k^2)A_k \\ &+ 2a^2A_{k-1} + a^2A_{k-2} - \sum_{i=0}^{k}(k-i+1) \\ &\times(\beta_{1,i-2} + 2\beta_{1,i-1} + \beta_{1i})A_{k-i+1} \\ &+ s^2\sum_{i=0}^{k}(k-i+1)(\beta_{2,i-2} + 2\beta_{2,i-1} + \beta_{2i})A_{k-i}\end{aligned} \tag{19}$$

Making use of Eq. (19), all coefficients A_k in series (17) can be derived by recursive computation. For $k = 0$, we can obtain the coefficient A_2, which is expressed by A_0 and A_1. For $k = 1$, we first carry out the coefficient A_3, which is expressed by A_0, A_1, and A_2. Second, submitting the preceding derived coefficient A_2 into the expression of A_3, then we can also obtain the relation expressed by A_0 and A_1. Continuing this recursive computation, similar expression of all coefficients A_k can be carried out. Therefore, the coefficient A_k can be briefly expressed as

$$A_k(s) = P_{1k}(s)\cdot A_0 + P_{2k}(s)\cdot A_1 \tag{20}$$

where the coefficient items $P_{1k}(s)$ and $P_{2k}(s)$ can be derived from Eq. (19). And then, the solution of Eq. (15) can be briefly expressed as

$$F(R,s) = A_0\sum_{k=0}^{\infty}P_{1k}(s)\cdot(R-1)^k + A_1\sum_{k=0}^{\infty}P_{2k}(s)\cdot(R-1)^k \tag{21}$$

where A_0 and A_1 are unknown constants. Substituting solution (21) into boundary conditions (16), the following algebraic equations can be obtained:

$$\begin{bmatrix} d_{11}(s) & d_{12}(s) \\ d_{21}(s) & d_{22}(s)\end{bmatrix}\begin{Bmatrix}A_0\\A_1\end{Bmatrix} = \begin{Bmatrix}-\frac{H_1\Theta_1}{s}\\ \frac{H_2\Theta_2}{s}\end{Bmatrix} \tag{22}$$

where

$$d_{11} = \sum_{k=0}^{\infty}[(k+1)P_{1,k+1}(s) - H_1P_{1k}(s)](R_a-1)^k$$
$$d_{12} = \sum_{k=0}^{\infty}[(k+1)P_{2,k+1}(s) - H_1P_{2k}(s)](R_a-1)^k$$
$$d_{21} = \sum_{k=0}^{\infty}[(k+1)P_{1,k+1}(s) - H_2P_{1k}(s)](R_b-1)^k$$
$$d_{22} = \sum_{n=0}^{\infty}[(k+1)P_{2,k+1}(s) - H_2P_{2k}(s)](R_b-1)^k$$

Making use of algebraic equation (22), we can easily carry out constants A_0 and A_1:

$$A_0 = \det\begin{bmatrix}-H_1\Theta_1 & d_{12}(s)\\ H_2\Theta_2 & d_{22}(s)\end{bmatrix}\Big/ s\cdot\det\begin{bmatrix}d_{11}(s) & d_{12}(s)\\ d_{21}(s) & d_{22}(s)\end{bmatrix} \tag{23a}$$

$$A_1 = \det\begin{bmatrix}d_{11}(s) & -H_1\Theta_1\\ d_{21}(s) & H_2\Theta_2\end{bmatrix}\Big/ s\cdot\det\begin{bmatrix}d_{11}(s) & d_{12}(s)\\ d_{21}(s) & d_{22}(s)\end{bmatrix} \tag{23b}$$

Substituting determined constants A_0 and A_1 from Eq. (23) into Eq. (21), we can finally obtain the solution of Eq. (15):

$$F(R,s) = \sum_{k=0}^{\infty}\frac{N_k(s)}{M(s)}(R-1)^k \tag{24}$$

where

$$N_k(s) = \det\begin{bmatrix}-H_1\Theta_1 & d_{12}(s)\\ H_2\Theta_2 & d_{22}(s)\end{bmatrix}P_{1k}(s) + \det\begin{bmatrix}d_{11}(s) & -H_1\Theta_1\\ d_{21}(s) & H_2\Theta_2\end{bmatrix}P_{2k}(s)$$
$$M(s) = s\cdot\det\begin{bmatrix}d_{11}(s) & d_{12}(s)\\ d_{21}(s) & d_{22}(s)\end{bmatrix}$$

To obtain the solution of time-dependent temperature, we should perform the inverse Laplace transformation on Eq. (24). Making use of residue theorem, Laplace inverse transformations of all coefficients of $(R-1)^k$ in Eq. (24) can be easily carried out and briefly expressed as

$$\Theta_{nm}(R,\tau)=\sum_{k=0}^{\infty}\left[\sum_{j=1}^{\infty}\frac{N_k(s_j)}{[\mathrm{d}M(s)/\mathrm{d}s]_{s=s_j}}e^{s_j\tau}\right](R-1)^k \tag{25}$$

where $s_0=0$ and s_j $(j=1,2,3,\ldots)$ are negative roots of transcendental equation $M(s)=0$.

Furthermore, substituting Eq. (25) into trigonometric series (11), the three-dimensional time-dependent temperature of the FGM panel can be calculated.

Analysis of Transient Thermomechanical Stress

Employing a Navier trigonometric series, solutions of Eq. (7) satisfying the boundary conditions (8) can be assumed as

$$U(R,\theta,Z,\tau)=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}U_{nm}(R,\tau)\cdot\sin(b\theta)\sin(aZ) \tag{26a}$$

$$V(R,\theta,Z,\tau)=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}V_{nm}(R,\tau)\cos(b\theta)\sin(aZ) \tag{26b}$$

$$W(R,\theta,Z,\tau)=\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}U_{nm}(R,\tau)\sin(b\theta)\cos(aZ) \tag{26c}$$

Substituting Eqs. (11) and (26) into Eq. (10) and boundary conditions (9), we have

$$\begin{aligned}&\left[\frac{\partial^2}{\partial R^2}+\left(\frac{Y'(R)}{Y(R)}+\frac{1}{R}\right)\frac{\partial}{\partial R}+\left(\frac{\mu}{1-\mu}\frac{1}{R}\frac{Y'(R)}{Y(R)}-\frac{1}{R^2}\right)\right.\\&\left.\quad-\frac{1-2\mu}{2-2\mu}\left(\frac{b^2}{R^2}+a^2\right)\right]U_{nm}\\&\quad-b\left[\frac{1}{2-2\mu}\frac{1}{R}\frac{\partial}{\partial R}+\frac{\mu}{1-\mu}\frac{1}{R}\frac{Y'(R)}{Y(R)}+\frac{(4\mu-3)}{2-2\mu}\frac{1}{R^2}\right]V_{nm}\\&\quad-a\left[\frac{1}{2-2\mu}\frac{\partial}{\partial R}+\frac{\mu}{1-\mu}\frac{Y'(R)}{Y(R)}\right]W_{nm}\\&\quad+\frac{1+\mu}{1-\mu}\left[\Omega(R)\cdot\frac{\partial}{\partial R}+\frac{[Y(R)\cdot\Omega(R)]'}{Y(R)}\right]\Theta_{nm}=0\end{aligned} \tag{27a}$$

$$\begin{aligned}&\left[\frac{\partial^2}{\partial R^2}+\left(\frac{Y'(R)}{Y(R)}+\frac{1}{R}\right)\frac{1}{R}\frac{\partial}{\partial R}-\frac{1}{R}\frac{Y'(R)}{Y(R)}-\frac{1}{R^2}-\frac{2-2\mu}{1-2\mu}\frac{b^2}{R^2}-a^2\right]V_{nm}\\&\quad+b\left[\frac{1}{1-2\mu}\frac{1}{R}\frac{\partial}{\partial R}+\frac{1}{R}\frac{Y'(R)}{Y(R)}+\frac{3-4\mu}{1-2\mu}\frac{1}{R^2}\right]U_{nm}-\frac{ab}{1-2\mu}\frac{1}{R}W_{nm}\\&\quad+b\frac{2+2\mu}{1-2\mu}\frac{\Omega(R)}{R}\Theta_{nm}=0\end{aligned} \tag{27b}$$

$$\begin{aligned}&a\left[\frac{1}{1-2\mu}\frac{\partial}{\partial R}+\frac{Y'(R)}{Y(R)}+\frac{1}{1-2\mu}\frac{1}{R}\right]U_{nm}-\frac{ab}{1-2\mu}\frac{1}{R}V_{nm}\\&\quad+\left[\frac{\partial^2}{\partial R^2}+\left(\frac{Y'(R)}{Y(R)}+\frac{1}{R}\right)\frac{\partial}{\partial R}-\frac{b^2}{R^2}-\frac{2-2\mu}{1-2\mu}a^2\right]W_{nm}\\&\quad+a\frac{2+2\mu}{1-\mu}\Omega(R)\cdot\Theta_{nm}=0\end{aligned} \tag{27c}$$

and

$$\begin{aligned}&(1-\mu)\frac{\mathrm{d}U_{nm}(R_a,\tau)}{\mathrm{d}R}-\frac{\mu b}{R_a}V_{nm}(R_a,\tau)+\frac{\mu}{R_a}U_{nm}(R_a,\tau)\\&\quad-\mu aW_{nm}(R_a,\tau)-(1+\mu)\Omega(R_a)\cdot\Theta_{nm}(R_a,\tau)=Q_{anm}\end{aligned} \tag{28a}$$

$$\frac{b}{R_a}U_{nm}(R_a,\tau)+\frac{\mathrm{d}V_{nm}(R_a,\tau)}{\mathrm{d}R}-\frac{V_{nm}(R_a,\tau)}{R_a}=0 \tag{28b}$$

$$\frac{\mathrm{d}W_{nm}(R_a,\tau)}{\mathrm{d}R}+aU_{nm}(R_a,\tau)=0 \tag{28c}$$

$$\begin{aligned}&(1-\mu)\frac{\mathrm{d}U_{nm}(R_b,\tau)}{\mathrm{d}R}-\frac{\mu b}{R_b}V_{nm}(R_b,\tau)+\frac{\mu}{R_b}U_{nm}(R_b,\tau)\\&\quad-\mu aW_{nm}(R_b,\tau)-(1+\mu)\Omega(R_b)\cdot\Theta_{nm}(R_b,\tau)=Q_{bnm}\end{aligned} \tag{28d}$$

$$\frac{b}{R_b}U_{nm}(R_b,\tau)+\frac{\mathrm{d}V_{nm}(R_b,\tau)}{\mathrm{d}R}-\frac{V_{nm}(R_b,\tau)}{R_b}=0 \tag{28e}$$

$$\frac{\mathrm{d}W_{nm}(R_b,\tau)}{\mathrm{d}R}+aU_{nm}(R_b,\tau)=0 \tag{28f}$$

According to the theory of series solving method of ordinary differential equations [17], if the coefficient items $Y'(R)/Y(R)$, $[\Omega(R)\cdot Y(R)]'/Y(R)$, and $\Omega(R)$ are analytical at point $R=1$ and can be expressed as Taylor's series in terms of $R-1$, then the solution of Eq. (27) can be expressed as the following Taylor's series:

$$U_{nm}(R,\tau)=\sum_{k=0}^{\infty}B_k(\tau)\cdot(R-1)^k \tag{29a}$$

$$V_{nm}(R,\tau)=\sum_{k=0}^{\infty}C_k(\tau)\cdot(R-1)^k \tag{29b}$$

$$W_{nm}(R,\tau)=\sum_{k=0}^{\infty}D_k(\tau)\cdot(R-1)^k \tag{29c}$$

Also, we expand the functions $Y'(R)/Y(R)$, $[\Omega(R)\cdot Y(R)]'/Y(R)$, and $\Omega(R)$ in Taylor's series at point $R=1$:

$$f_3(R)=\frac{1}{Y(R)}\frac{\mathrm{d}Y(R)}{dR}=\sum_{k=0}^{\infty}f_{3k}\cdot(R-1)^k \tag{30a}$$

$$f_4(R)=\frac{1}{Y(R)}\frac{\mathrm{d}[\Omega(R)\cdot Y(R)]}{dR}=\sum_{k=0}^{\infty}f_{4n}k\cdot(R-1)^k \tag{30b}$$

$$f_5(R)=\Omega(R)=\sum_{k=0}^{\infty}f_{5k}\cdot(R-1)^k \tag{30c}$$

where

$$f_{3k}=\frac{1}{k!}f_3^{(k)}(1),\qquad f_{4k}=\frac{1}{k!}f_4^{(k)}(1),\qquad f_{5k}=\frac{1}{k!}f_5^{(k)}(1)$$

Furthermore, the solution of Eq. (25) can be briefly expressed as

$$\Theta_{nm}(R,\tau)=\sum_{k=0}^{\infty}G_k(\tau)\cdot(R-1)^k \tag{31}$$

where

$$G_k(\tau)=\sum_{j=1}^{\infty}\frac{N_k(s_j)}{[\mathrm{d}M(s)/\mathrm{d}s]_{s=s_j}}e^{s_j\tau}$$

Substituting series (29–31) into Eq. (27), employing the Abel principle of series multiplication, and comparing the coefficients of $(R-1)^k$, the following recurrence relations can be obtained:

$$\begin{aligned}&-(k+1)(k+2)B_{k+2}=(2k+1)(k+1)B_{k+1}\\&+\left(k^2-a^2-\frac{1-2\mu}{2-2\mu}b^2-1\right)B_k-2a^2B_{k-1}-a^2B_{k-2}\\&+\frac{\mu}{1-\mu}\sum_{i=0}^{k}(f_{3,i-1}+f_{3i})B_{k-i}+\sum_{i=0}^{k}(f_{3,i-2}+2f_{3,i-1}+f_{3i})B_{k-i}\\&-\frac{b}{2-2\mu}(k+1)C_{k+1}-\frac{k+4\mu-3}{2-2\mu}bC_k\\&-\frac{b\mu}{1-\mu}\sum_{i=0}^{k}(f_{3,i-1}+f_{3i})C_{k-i}\\&-\frac{a}{2-2\mu}[(k+1)D_{k+1}+2kD_k+(k-1)D_{k-1}]\\&-\frac{a\mu}{1-\mu}\sum_{i=0}^{k}(f_{3,i-2}+2f_{3,i-1}+f_{3i})D_{k-i}\\&+\frac{1+\mu}{1-\mu}\sum_{i=0}^{k}(k-i+1)(f_{5,i-2}+2f_{5,i-1}+f_{5i})G_{k-i+1}\\&+\frac{1+\mu}{1-\mu}\sum_{i=0}^{k}(f_{4,i-2}+2f_{4,i-1}+f_{4i})G_{k-i}\end{aligned}\tag{32a}$$

$$\begin{aligned}&-(k+1)(k+2)C_{k+2}=\frac{b}{1-2\mu}[(k+1)B_{k+1}+(k+3-4\mu)B_k]\\&+(2k+1)(k+1)C_{k+1}+\left(k^2-a^2-\frac{2-2\mu}{1-2\mu}b^2-1\right)C_k\\&-2a^2C_{k-1}-a^2C_{k-2}-\frac{ab}{1-2\mu}(D_k+D_{k-1})\\&+\sum_{i=0}^{k}(f_{3,i-2}+f_{3,i-1})C_{k-i}+b\sum_{i=0}^{k}(f_{3,i-1}+f_{3i})B_{k-i}\\&+b\frac{2+2\mu}{1-2\mu}\sum_{i=0}^{k}(f_{5,i-1}+f_{5i})G_{k-i}\end{aligned}\tag{32b}$$

$$\begin{aligned}&-(k+1)(k+2)D_{k+2}=\frac{a}{1-2\mu}[(k+1)B_{k+1}+(2k+1)B_k+kB_{k-1}]\\&-\frac{ab}{1-2\mu}(C_k+C_{k-1})+(2k+1)(k+1)D_{k+1}\\&+(k^2-b^2)D_k-\frac{2-2\mu}{1-2\mu}a^2(D_k+2D_{k-1}+D_{k-2})\\&+\sum_{i=0}^{k}(f_{3,i-2}+2f_{3,i-1}+f_{3i})D_{k-i}\\&+a\sum_{i=0}^{k}(f_{3,i-2}+2f_{3,i-1}+f_{3i})B_{k-i}\\&+a\frac{2+2\mu}{1-\mu}\sum_{i=0}^{k}(f_{5,i-2}+2f_{5,i-1}+f_{5i})G_{k-i}\end{aligned}\tag{32c}$$

Making use of Eq. (32), all coefficients B_k, C_k, and D_k in series (29) can be obtained by recursive computation. For $k=0$, we can derive the coefficients B_2, C_2, and D_2, which are expressed by B_0, B_1, C_0, C_1, D_0, D_1, and items related to G_k. For $k=1$, we first carry out the coefficients B_3, C_3, and D_3, which are expressed by B_0, B_1, B_2, C_0, C_1, C_2, D_0, D_1, D_2, and items related to G_k. Second, submitting the preceding derived B_2, C_2, and D_2 into the expression of B_3, C_3, and D_3, then we can also obtain three relations expressed by B_0, B_1, C_0, C_1, D_0, D_1, and items related to G_k. Continuing this recursive computation, similar expressions of all coefficients B_k, C_k, and D_k can be carried out. Therefore, the coefficients B_k, C_k, and D_k can be briefly expressed as

$$B_k(\tau)=L_1^kB_0+L_2^kB_1+L_3^kC_0+L_4^kC_1+L_5^kD_0+L_6^kD_1+L_7^k\tag{33a}$$

$$C_k(\tau)=P_1^kB_0+P_2^kB_1+P_3^kC_0+P_4^kC_1+P_5^kD_0+P_6^kD_1+P_7^k\tag{33b}$$

$$D_k(\tau)=Q_1^kB_0+Q_2^kB_1+Q_3^kC_0+Q_4^kC_1+Q_5^kD_0+Q_6^kD_1+Q_7^k\tag{33c}$$

where the coefficients L_i^k, P_i^k, and Q_i^k can be derived from Eq. (32). So the solutions of Eq. (27) can be briefly expressed as

$$\begin{aligned}U_{nm}(R,\tau)&=B_0\sum_{k=1}^{\infty}L_1^k(R-1)^k+B_1\sum_{k=1}^{\infty}L_2^k(R-1)^k\\&+C_0\sum_{k=1}^{\infty}L_3^k(R-1)^k+C_1\sum_{k=1}^{\infty}L_4^k(R-1)^k+D_0\sum_{k=1}^{\infty}L_5^k(R-1)^k\\&+D_1\sum_{k=1}^{\infty}L_6^k(R-1)^k+\sum_{k=1}^{\infty}L_7^k(R-1)^k\end{aligned}\tag{34a}$$

$$\begin{aligned}V_{nm}(R,\tau)&=B_0\sum_{k=1}^{\infty}P_1^k(R-1)^k+B_1\sum_{k=1}^{\infty}P_2^k(R-1)^k\\&+C_0\sum_{k=1}^{\infty}P_3^k(R-1)^k+C_1\sum_{k=1}^{\infty}P_4^k(R-1)^k+D_0\sum_{k=1}^{\infty}P_5^k(R-1)^k\\&+D_1\sum_{k=1}^{\infty}P_6^k(R-1)^k+\sum_{k=1}^{\infty}P_7^k(R-1)^k\end{aligned}\tag{34b}$$

$$\begin{aligned}W_{nm}(R,\tau)&=B_0\sum_{k=1}^{\infty}Q_1^k(R-1)^k+B_1\sum_{k=1}^{\infty}Q_2^k(R-1)^k\\&+C_0\sum_{k=1}^{\infty}Q_3^k(R-1)^k+C_1\sum_{k=1}^{\infty}Q_4^k(R-1)^k+D_0\sum_{k=1}^{\infty}Q_5^k(R-1)^k\\&+D_1\sum_{k=1}^{\infty}Q_6^k(R-1)^k+\sum_{k=1}^{\infty}Q_7^k(R-1)^k\end{aligned}\tag{34c}$$

where B_0, B_1, C_0, C_1, D_0, and D_1 are unknown constants and can be determined by substituting Eq. (34) into boundary conditions (28).

Substituting solutions (34) into Eq. (26), we can obtain the three-dimensional solutions of displacement of the FGM panel. Moreover, substituting Eq. (26) into Eq. (6) and then into Eq. (5), we can obtain the three-dimensional solutions of transient thermomechanical stresses:

$$\begin{aligned}\Sigma_r(R,\theta,Z,\tau)&=\frac{Y(R)}{(1+\mu)(1-2\mu)}\\&\times\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left\{\left[(1-\mu)\frac{\mathrm{d}}{\mathrm{d}R}+\frac{\mu}{R}\right]U_{nm}(R,\tau)-\frac{b\mu}{R}V_{nm}(R,\tau)\right.\\&\left.+a\mu W_{nm}(R,\tau)-(1+\mu)\Omega(R)\cdot\Theta_{nm}(R,\tau)\right\}\\&\times\sin(b\theta)\sin(aZ)\end{aligned}\tag{35a}$$

$$\Sigma_\theta(R,\theta,Z,\tau)=\frac{Y(R)}{(1+\mu)(1-2\mu)}\times\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left\{\left[\mu\frac{\mathrm{d}}{\mathrm{d}R}+\frac{1-\mu}{R}\right]U_{nm}(R,\tau)-\frac{(1-\mu)b}{R}V_{nm}(R,\tau)+a\mu W_{nm}(R,\tau)-(1+\mu)\Omega(R)\cdot\Theta_{nm}(R,\tau)\right\}\times\sin(b\theta)\sin(aZ) \tag{35b}$$

$$\Sigma_z(R,\theta,Z,\tau)=\frac{Y(R)}{(1+\mu)(1-2\mu)}\times\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left\{\left[\mu\frac{\mathrm{d}}{\mathrm{d}R}+\frac{1-\mu}{R}\right]U_{nm}(R,\tau)-\frac{\mu b}{R}V_{nm}(R,\tau)+a(1-\mu)W_{nm}(R,\tau)-(1+\mu)\Omega(R)\cdot\Theta_{nm}(R,\tau)\right\}\times\sin(b\theta)\sin(aZ) \tag{35c}$$

$$\Sigma_{r\theta}(R,\theta,Z,\tau)=\frac{Y(R)}{2(1+\mu)}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left\{\frac{b}{R}U_{nm}(R,\tau)+\left[\frac{\mathrm{d}}{\mathrm{d}R}-\frac{1}{R}\right]V_{nm}(R,\tau)\right\}\cos(b\theta)\sin(aZ) \tag{35d}$$

$$\Sigma_{rz}(R,\theta,Z,\tau)=\frac{Y(R)}{2(1+\mu)}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left\{aW_{nm}(R,\tau)+\frac{\mathrm{d}}{\mathrm{d}R}U_{nm}(R,\tau)\right\}\sin(b\theta)\cos(aZ) \tag{35e}$$

$$\Sigma_{\theta z}(R,\theta,Z,\tau)=\frac{Y(R)}{2(1+\mu)}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left\{aW_{nm}(R,\tau)+\frac{b}{R}V_{nm}(R,\tau)\right\}\cos(b\theta)\cos(aZ) \tag{35f}$$

Results and Discussion

Here, we consider a molybdenum/mullite FGM cylindrical panel with the geometric parameters $R_1=0.9$, $R_2=1.1$, $L=5$, and $\theta_0=\pi/3$, as shown in Fig. 1. The inner and outer surfaces of the panel are pure mullite and composite of molybdenum/mullite, respectively. Both molybdenum and mullite vary continuously from inner to outer surfaces of the panel. E_0, α_0, λ_0, κ_0, and μ of mullite taken from Awaji and Sivakuman [18] are 225 GPa, 4.8×10^{-6} K^{-1}, 5.9 $\mathrm{W(mK)}^{-1}$, 2.8×10^{-6} $\mathrm{m^2s^{-1}}$, and 0.3, respectively. It is assumed that the variations of material properties through the thickness of the panel obey the following exponential laws [19]:

$$Y(R)=\exp[m_1(R-R_1)] \tag{36}$$

$$\Omega(R)=\exp[m_2(R-R_1)] \tag{36b}$$

$$\Lambda(R)=\exp[m_3(R-R_1)] \tag{36c}$$

$$K(R)=\exp[m_4(R-R_1)] \tag{36d}$$

where m_1, m_2, m_3, and m_4 are material constants. The heat transfer coefficients on the inner and outer surfaces of the panel are $H_1=H_2=30$, respectively. Temperatures of internal and external surrounding media are assumed as

$$\Theta_a(\theta,\ z)=\sin(3\theta)\cdot\sin(\pi z/6)\,,\qquad \Theta_b(\theta,\ z)=0$$

and mechanical loads applied on the inner and outer surfaces are assumed as

$$Q_a(\theta,\ z)=0,\qquad Q_b(\theta,\ z)=0$$

First, we assume the material constants of material property variations to be $m_1=2.0$, $m_2=0.3$, $m_3=3.0$, and $m_4=2.0$. Based on the preceding assumption, all numerical results are presented in Figs. 2–8. For the sake of brevity, only radial distributions are graphically presented here.

Figure 2 shows the time-dependent radial distribution of dimensionless temperature. It is seen that temperatures of the panel increase with time proceeding and finally approach their maximum value at steady state. At any fixed time, the temperature gradually decreased from the inner surface to the outer surface in the radial direction. Because of the nonhomogeneity of FGM, these radial distributions of temperature always vary with nonlinear form throughout the whole heating-time process.

The radial distribution of dimensionless transient radial thermomechanical stress in the FGM panel is shown in Fig. 3. It is seen that the radial stress Σ_r is compressive for the assumed FGM

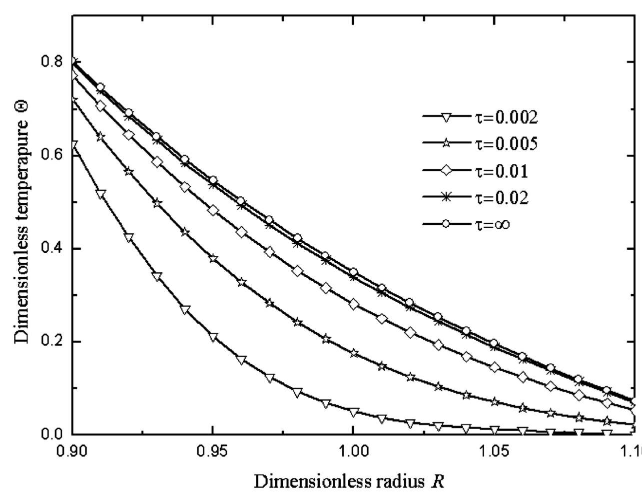

Fig. 2 Radial distribution of dimensionless time-dependent temperature; $\theta=\pi/6$ and $Z=3.0$.

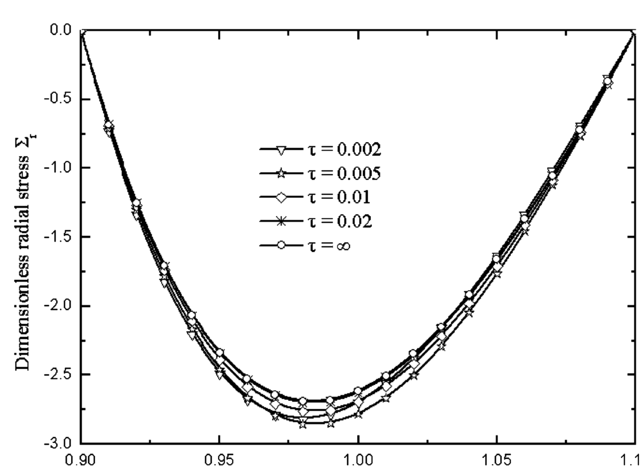

Fig. 3 Radial distributions of dimensionless transient thermomechanical radial stress Σ_r; $\theta=\pi/6$ and $Z=3.0$.

panel. Magnitude of the radial thermomechanical stress does not always increase with time; the maximum compressive stress occurs at a transient state but not at the steady state.

Figures 4 and 5 show the time-dependent radial distributions of circumferential and axial thermomechanical stresses Σ_θ and Σ_z. It is seen that due to the assumed thermomechanical loads, both the circumferential and axial stresses are tensile on the inner surface and compressive on the outer surface. Magnitudes of the circumferential and axial stresses increase with time proceeding, and their maximum values occur at steady state.

Figures 6–8 show the time-dependent radial distributions of three thermomechanical shear stresses (i.e., $\Sigma_{r\theta}$ $\Sigma_{\theta z}$, and Σ_{rz}). It is seen that magnitudes of these three shear stresses are far smaller than the other three normal stresses. These magnitudes also increase with heating time proceeding and approach their maximum values at steady state.

Additionally, we study the effects of material graded factor on the temperature and thermomechanical stresses using the present method. For the sake of brevity, the variations of material properties are assumed to obey the rule $m_1 = m_2 = m_3 = m_4 = \alpha$, and only effects at steady state on temperature and three normal thermomechanical stresses are graphically presented.

Figure 9 shows the effects of material graded factor α on steady-state temperature. It is seen that the material graded factor presents a significant effect on the temperature. According to the exponential model, increasing material graded factor α means an increment in the volume fraction of mullite. Because of the higher conductive capacity of mullite, the conductive capacity of the functionally graded material is also increased with factor α. For the same heat flux, the temperature of the panel is otherwise decreased with factor α.

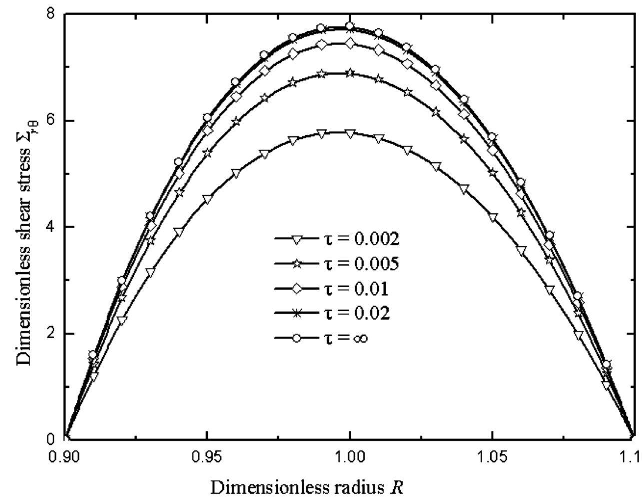

Fig. 6 Radial distributions of dimensionless transient thermomechanical shear stress $\Sigma_{r\theta}$; $\theta = 0$ and $Z = 3.0$.

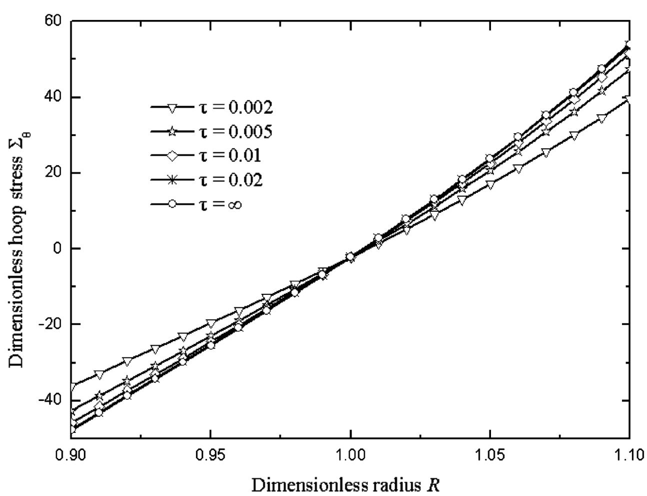

Fig. 4 Radial distributions of dimensionless transient thermomechanical circumferential stress; $\theta = \pi/6$ and $Z = 3.0$.

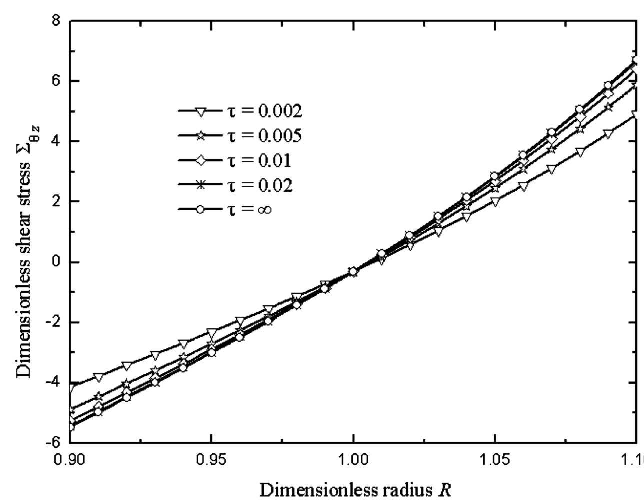

Fig. 7 Radial distributions of dimensionless transient thermomechanical shear stress $\Sigma_{\theta z}$; $\theta = 0$ and $Z = 6.0$.

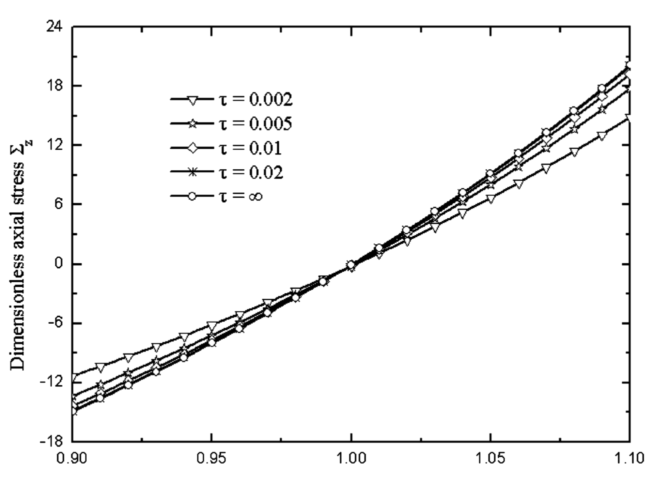

Fig. 5 Radial distributions of dimensionless thermomechanical axial stress; $\theta = \pi/6$ and $Z = 3.0$.

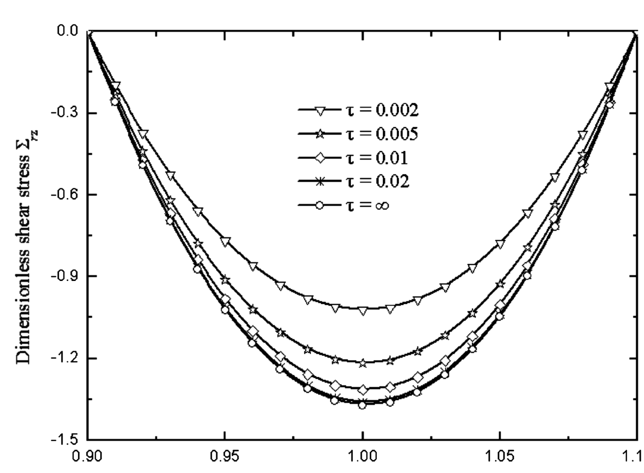

Fig. 8 Radial distributions of dimensionless transient thermomechanical shear stress Σ_{rz}; $\theta = \pi/6$ and $Z = 6.0$.

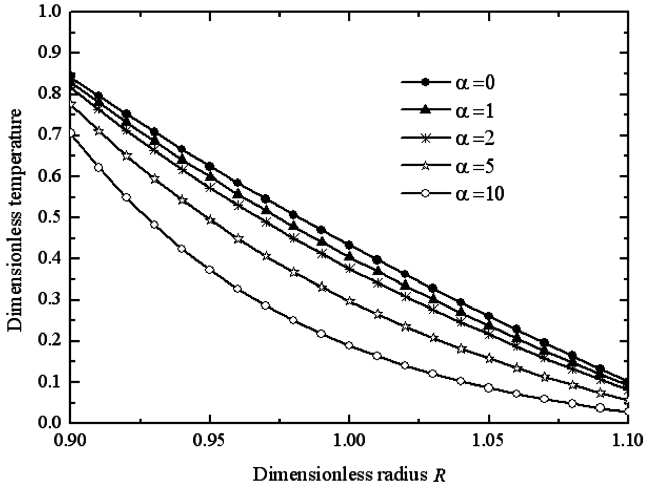

Fig. 9 Effects of material graded factor α on temperature.

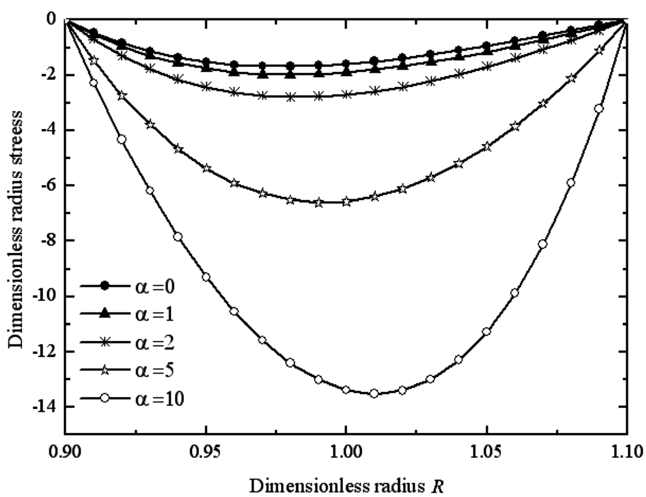

Fig. 10 Effects of material graded factor α on thermomechanical radial stress.

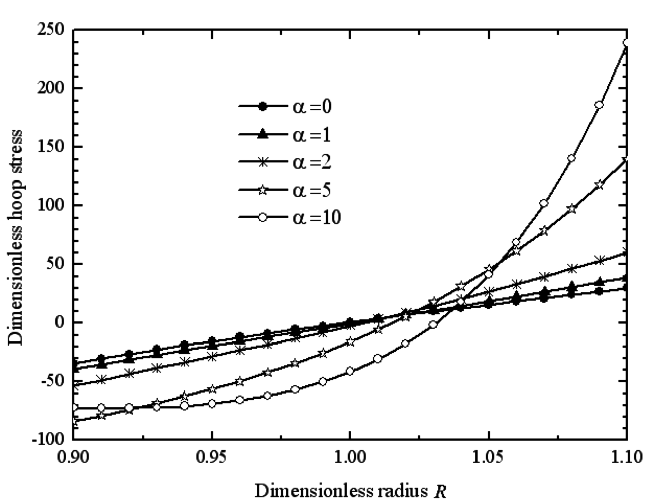

Fig. 11 Effects of material graded factor α on thermomechanical circumferential stress.

Figures 10–12 show the effects of material graded factor α on three normal thermomechanical stresses at steady state. It is seen that the material graded factor α dramatically affects these three normal thermomechanical stresses. Unlike the effect on temperature, these three normal thermomechanical stresses increase with factor α.

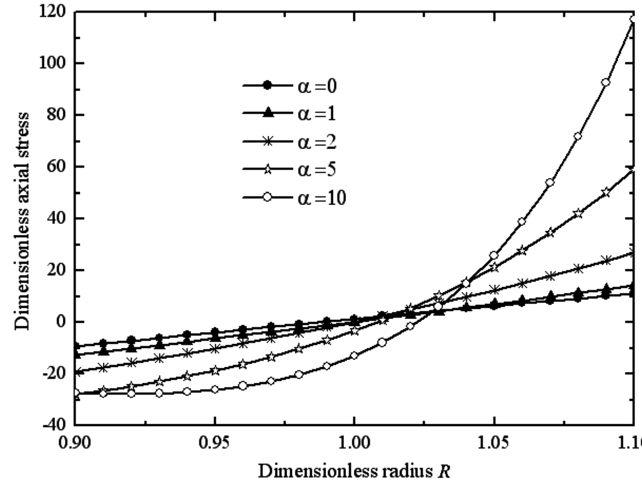

Fig. 12 Effects of material graded factor α on thermomechanical axial stress.

Because of the higher thermal expansion coefficients of mullite, the thermal expansion coefficient of functionally graded material increases with factor α. Although the temperature decreased with factor α, due to the much higher thermal expansion coefficient, thermomechanical stresses of the FGM panel increased with the factor α.

Conclusions

Analysis of three-dimensional transient thermomechanical problem of a functionally graded cylindrical panel is carried out in this paper. Using Laplace transformation and the series solving method of ordinary differential equation(s), analytical solutions of time-dependent temperature and thermomechanical stresses are obtained. As an example, a molybdenum/mullite functionally graded cylindrical panel under assumed transient thermomechanical loading is considered and the results are graphically presented. The effects of material graded factor on the thermomechanical stresses are also evaluated.

The advantage of the present method is its applicability to any material model suggested for functionally graded materials, and the continuous variation of material properties can be fully included in the solutions. It should be emphasized that the trigonometric series used in this paper are only suitable for the thermal and mechanical boundary conditions assumed in the paper. For other boundary conditions, one should choose other suitable forms of the trigonometric series.

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A. Roy
Associate Editor